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# NONISOTHERMAL MOTION OF A RAREFIED GAS IN A SHORT PLANAR CHANNEL OVER A WIDE RANGE OF KNUDSEN NUMBER

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Results are presented from a calculation of two-dimensional nonisothermal motion of a rarefied gas in a short planar channel. Calculations were performed for two different temperature distributions along the channel wall. Kinetic coefficients are calculated and the Onsager relationships are verified. Thermomolecular pressure difference indices are found.

A brief overview of studies on motion of rarefied gases in finite length channels was presented in [1], whence it follows that practically all studies in that field have been limited to consideration of isothermal gas flow. Until the present nonisothermal flow has been considered only in an infinite channel. Thus, the study of nonisothermal gas flows in finite channels is of practical interest.

We will consider a planar channel of length l, height a, and infinite in the z-direction, as shown in Fig. 1, joining two infinite vessels containing one and the same gas. At sufficient removal from the channel within the vessels the gas is maintained under equilibrium conditions at pressures  $P_1$  and  $P_2$  and temperatures  $T_1$  and  $T_2$ . The equilibrium distribution functions have absolute Maxwell forms:

$$f_{i} = \frac{P_{i}}{kT_{i}} \left(\frac{m}{2\pi kT_{i}}\right)^{3/2} \exp\left(-\frac{mv^{2}}{2kT_{i}}\right), \quad i = 1, 2.$$
(1)

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Fig. 1. Problem geometry.

The walls of each vessel (i.e., the barrier) are at temperatures  $T_1$  and  $T_2$ , respectively. Along the channel walls there is a temperature distribution  $T_w(x) = T_1 + \tau_w(x)(T_2 - T_1)$ , where  $\tau_w(x)$  is a specified function. It is assumed that upon interaction with the walls all molecules reflect diffusely, and that pressure and temperature heads are small,  $|P_2 - P_1|/P_1 \ll 1$ ,  $|T_2 - T_1|/T_1 \ll 1$ . It is necessary to find the flow field and mass and heat fluxes at arbitrary Knudsen numbers. In parallel with this task, which we will refer to below as problem No. 1, we will consider the case where preinput regions are absent, and on the channel faces conditions of the following form are given: at x = 0,  $c_x \ge 0$ ,  $f = f_1$ ; at x = L,  $c_x \le 0$ ,  $f = f_2$ . Such a formulation will be referred to as problem No. 2 below. Comparison of the results of problems Nos. 1 and 2 will permit determining the role of the preinput regions in gas flow formation.

For an exact solution of the problem formulated Boltzmann's equation must be used, but for arbitrary gas rarefaction the complex structure of the collision integral will not permit such an approach. Therefore, a third order model equation (S-model) was used, insuring an adequate description of heat and mass transport simultaneously. This equation has the form [2]:

$$\mathbf{v} \frac{\partial f}{\partial \mathbf{r}} = \frac{P}{\eta} (f^+ - f),$$

$$f^+ = f^0 \left[ 1 + \frac{4}{15} \mathbf{S} \cdot \mathbf{W} \left( W^2 - \frac{5}{2} \right) \right]; \quad \mathbf{S} = \frac{1}{n} \int \mathbf{W} W^2 f d\mathbf{v},$$

$$f^0 = n \left( \frac{m}{2\pi kT} \right)^{3/2} \exp\left( -W^2 \right), \quad \mathbf{W} = \sqrt{\frac{m}{2kT}} (\mathbf{v} - \mathbf{u}),$$

$$n = \int f d\mathbf{v}, \quad n\mathbf{u} = \int \mathbf{v} f d\mathbf{v}, \quad P = nkT, \quad \eta = \frac{1}{2} nmv_T \lambda,$$

$$T = \frac{2}{3k} \int \frac{mv^2}{2} f d\mathbf{v}, \quad v_T = \left( \frac{8kT}{\pi m} \right)^{1/2},$$
(2)

where  $\lambda$  is the molecular free path length.

We introduce the following dimensionless quantities:  $\mathbf{c} = (m/2kT_1)^{1/2}\mathbf{v}$ ,  $\mathbf{u}' = (m/2kT_1)^{1/2}\mathbf{u}$  for velocities of molecules and gas, respectively,  $\mathbf{q}' = [0.5n_1m(2kT_1/m)^{3/2}]^{-1}\mathbf{q}$  for the heat flux density, x' = x/a, y' = y/a for coordinates,  $J'_k = (m/2kT_1)^{1/2}/(kn_1a)J_k$ ,  $\Lambda'_{kl} = (m/2kT_1)^{1/2}/(kn_1a)\Lambda_{kl}$  for thermodynamic fluxes and kinetic coefficients. In further calculations we will omit the primes from symbols for dimensionless quantities.

In the case of small pressure and temperature heads the unknown distribution function can be written in the form

 $f(\mathbf{r}, \mathbf{c}) = f_1 [1 + h(\mathbf{r}, \mathbf{c})], \quad |h| \ll 1,$  (3)

where  $f_1$  is defined by Eq. (1). Substituting Eq. (3) into Eq. (2), we obtain a linearized S-model

$$\mathbf{c} \; \frac{\partial h}{\partial \mathbf{r}} = L_s h, \tag{4}$$

$$L_{s}h = \delta \left[\vartheta + 2\mathbf{c} \cdot \mathbf{u} + \left(\frac{4}{15} \mathbf{q} \cdot \mathbf{c} + \tau\right) \left(c^{2} - \frac{5}{2}\right) - h\right], \qquad (4)$$

where  $\delta = \sqrt{\pi} a/2\lambda$  is the rarefaction parameter,

$$[\vartheta; \mathbf{u}; \tau, \mathbf{q}] = \int f_1 h \left[ \frac{2}{3} c^2; \mathbf{c}; \frac{2}{3} c^2 - 1; \mathbf{c} \left( c^2 - \frac{5}{2} \right) \right] d\mathbf{c}.$$

Here  $\vartheta = (P - P_1)/P_1$ ;  $\tau = (T - T_1)/T_1$ . To solve kinetic equation (4) we use the integralmoment method, with the aid of which we obtain a system of integral equations for the moments of the distribution functions:

$$M_{i}(\mathbf{r}) = \frac{\delta}{\pi} \sum_{j=1}^{6} \int_{0}^{2\pi} \int_{0}^{s_{0}} K_{ij}(\mathbf{r}, \mathbf{r}') M_{j}(\mathbf{r}') ds d\phi + \frac{1}{\pi} \int_{0}^{2\pi} \Phi_{i}(\mathbf{r}) d\phi,$$
(5)

where  $1 \leq i \leq 6$ ,  $\mathbf{r}' = (x', y') = (x - s \cos \varphi, y - s \sin \varphi)$ ,  $s_0$  is the distance from the observation point to the boundary of the flow field in the direction  $-(c_x + c_y)$ . Here we have introduced the notation  $M_1 = \vartheta(\mathbf{r})$ ,  $M_2 = u_x(\mathbf{r})$ ,  $M_3 = u_y(\mathbf{r})$ ,  $M_4 = \tau(\mathbf{r})$ ,  $M_5 = q_x(\mathbf{r})$ ,  $M_6 = q_y(\mathbf{r})$ . The procedure of deriving integral equations of the form of Eq. (5) was described in detail in [1], so that the cumbersome expression for  $K_{ij}$  and  $\Phi_i$  will be omitted here. The Krylov-Bogolyubov method [3], easily extensible to a system of two-dimensional integral equations, was used to solve the system.

In numerical calculations two forms of temperature distribution along the channel walls were used:

$$\tau_w = \frac{x}{L}, \quad \tilde{\tau}_w = \begin{cases} 0, & x < L/2 \\ 1, & x \geqslant L/2, \quad L = l/a. \end{cases}$$

In the first case the channel wall temperature changes linearly from  $T_1$  to  $T_2$ , while in the second the temperature of one half of the channel is equal to  $T_1$ , that of the other,  $T_2$ . In both problems (Nos. 1 and 2) one can introduce three independent thermodynamic forces  $X_P = \Delta P/P_1$ ,  $X_T = \Delta T/T_1$ ,  $X_T^T = \Delta T/T_1$ . Here  $\Delta T$  corresponds to the temperature head with distribution  $\tau_W$ , and  $\Delta T$  to the distribution  $\tau_W$ . As was shown in [4], the thermodynamic fluxes in problems Nos. 1 and 2 have one and the same form and with consideration of the scale factors introduced can be written in the following manner:

$$J_{p} = -\int_{-1/2}^{1/2} u_{x} dy,$$

$$J_{T} = -\int_{-1/2}^{1/2} q_{x} dy|_{x=L} - 2\int_{0}^{L} q_{y} \frac{x}{L} dx \Big|_{y=1/2},$$

$$J_{\widetilde{T}} = -\int_{-1/2}^{1/2} q_{x} dy|_{x=L} - 2\int_{L/2}^{L} q_{y} dx|_{y=1/2} = -\int_{-1/2}^{1/2} q_{y} dy|_{x=L/2}.$$
(6)

To transform the expressions  $J_T^{\tilde{T}}$  to their final form we use the law of conservation of energy in the right half of the channel. The quantity  $J_P$  has the sense of a volume mass flux per unit channel width. The quantities  $J_T$  and  $J_T^{\tilde{T}}$  consist of two terms: The first corresponds to the thermal flux through the channel input section, while the second is related to the thermal flux through the channel wall, integrated over the entire length with a weight dependent on the temperature distribution. Only with such a choice of thermodynamic fluxes will the entropy production have the form  $\sigma = \sum_k J_k X_k$ , k = P, T,  $\tilde{T}$ , which is obtained in linear nonequilibrium thermodynamics [5].

We expand the moments of the distribution function into components:

$$\vartheta = \vartheta^{P} X_{P} + \vartheta^{T} X_{T} + \vartheta^{\tilde{T}} X_{\tilde{T}}, \quad \mathbf{u} = \mathbf{u}^{P} X_{P} + \mathbf{u}^{T} X_{T} + \mathbf{u}^{\tilde{T}} X_{\tilde{T}},$$

$$\tau = \tau^{P} X_{P} + \tau^{T} X_{T} + \tau^{\tilde{T}} X_{\tilde{T}}, \quad \mathbf{q} = \mathbf{q}^{P} X_{P} + \mathbf{q}^{T} X_{T} + \mathbf{q}^{\tilde{T}} X_{\tilde{T}}.$$
(7)

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Substituting Eq. (7) into Eq. (6), we obtain the linear dependence of the thermodynamic fluxes  $J_n$  on the thermodynamic forces  $X_k$ :

$$J_n = \sum_k \Lambda_{nk} X_k, \quad n, \ k = P, \ T, \ \tilde{T}.$$

The kinetic coefficients  $\Lambda_{nk}$  will have a form analogous to the corresponding flux  $J_n$ , where in place of the moments u and q there will appear their components with subscript k (k = P, T, T). For the nondiagonal kinetic coefficients  $\Lambda_{kn}$  the Onsager relationships

$$\Lambda_{PT} = \Lambda_{TP}, \quad \Lambda_{P\tilde{T}} = \Lambda_{\tilde{T}P}, \quad \Lambda_{T\tilde{T}} = \Lambda_{\tilde{T}T} \tag{8}$$

are valid. These relationships can be obtained from general considerations of the linear thermodynamics of irreversible processes [5]. It was shown in [4] that for the system considered relationships (8) are the consequence of the self-conjugate nature of the linearized Boltzmann collision operator and the reciprocity of the scattering integrands for gas molecules on the channel and vessel walls. Diffuse scattering insures satisfaction of the second condition. In the kinetic equation in place of the exact collision operator we use the model  $L_s$ . It can easily be shown that the operator  $L_s$  is also self-conjugate, so that its use in place of the exact operator insures satisfaction of Eq. (8).

We will consider briefly the physical meaning of each kinetic coefficient. The quantity  $\Lambda_{\rm PP}$  corresponds to mass transport under the influence of the pressure head or beyond the Poiseuille flow. The coefficients  $\Lambda_{\rm TT}$  and  $\Lambda_{\rm TT}^{-}$  correspond to heat transport under the action of the temperature head for linear and steplike distributions along the channel wall, respectively. The coefficients  $\Lambda_{\rm PT}$  and  $\Lambda_{\rm PT}^{-}$  correspond to mass transport under the influence of the temperature head, i.e., thermocreep, for the various temperature distributions on the channel walls.

The coefficients  $\Lambda_{TP}$  and  $\Lambda_{TP}^{\sim}$  correspond to heat transport under the influence of the pressure head, i.e., to the mechanocaloric effect, and according to Eq. (8), are equal to  $\Lambda_{PT}$  and  $\Lambda_{PT}^{\sim}$ , respectively. We note that the coefficients  $\Lambda_{TP}$  and  $\Lambda_{TP}^{\sim}$  can be calculated in



Fig. 3. Velocity profiles at L = 5 and  $\delta$  = 2 for problem No. 1 in various sections. Solid line, x = 0; dashes, x = L/2.

terms of one and the same thermal flux density field  $q^P$ . In both cases the thermal flux is taken in the channel input section, while the flux density on the lateral surface is integrated with the linear or stepped temperature distribution, respectively. The stepped distribution  $\tau_W$  permits expression of the coefficient  $\Lambda_{TP}^{TP}$  in terms of the thermal flux in the mean channel section.

The sense of  $\Lambda_{T\widetilde{T}}$  and  $\Lambda_{\widetilde{T}T}$  is that one can be calculated in terms of the thermal flux density field created by the temperature head with distribution  $\tau_w$ , but the integral over the lateral surface is taken with a weight  $\tilde{\tau}_w$ . The other coefficient, on the other hand, is calculated in terms of the field created by  $\Delta \tilde{T}$  with distribution  $\tilde{\tau}_w$ , but weight  $\tau_w$ .

The calculations were performed to within an accuracy of 2%, for two channel lengths, L = 1 and 5, for a range of numbers  $\delta$  from 0.02 to 2 in problem No. 1 and 0.02 to 8 for problem No. 2. The calculation accuracy was monitored by comparing results of calculations on different grids. In all cases reciprocity relationships (8) were satisfied within the limits of experimental error, as well as the conservation laws for particles and energy.

Figure 2 shows the dependence of kinetic coefficients on the number  $\delta$ . All coefficients except  $\Lambda_{PP}$  decrease in modulus with increase in  $\delta$ . It is evident from Fig. 2a that at short lengths L and large numbers  $\delta$  the coefficient  $\Lambda_{PP}$  of problem No. 1 differs greatly from the same coefficient of problem No. 2. For the kinetic coefficient  $\Lambda_{PT}$  corresponding to thermocreep with linear temperature distribution the difference between the two problems is insignificant. For example, at L = 5 and  $\delta$  = 2 this difference comprises 5%. The lifference between the coefficients  $\Lambda_{PT}$  in problems Nos. 1 and 2 is somewhat larger, comprising 16% for the same L and  $\delta$ .

It is interesting to compare thermocreep values for each problem separately for the different temperature distributions along the channel wall  $\tau_W$  and  $\tilde{\tau}_W$ . In problem No. 1 this difference proved to be quite small: at L = 5 and  $\delta$  = 2 it comprised 3%, which exceeds the calculation uncertainty only insignificantly. This might raise doubts as to the existence of a dependence of thermocreep on the channel wall temperature distribution. If we analyze the structure of the coefficient  $\Lambda_{TP}$ , which according to Eq. (8) is equal to  $\Lambda_{PT}$ , then it is evident that in the second term of the integral we have the product of the flux on the wall  $q_y P |_{y=1/2}$  and the temperature distribution  $\tau_W$ . In the absence of heat exchange between the gas and wall  $q_y P |_{y=1/2} = 0$  the second term vanishes, and thus, so does the thermocreep  $L_{PT}$  produced by  $\tau_W$ . But, as analysis of numerical results shows, in the presence of such heat exchange the contribution of the second term to kinetic coefficient  $\Lambda_{TP}$  in problem No. 1 comprises 8% at L = 5 and  $\delta$  = 2, which significantly exceeds the calculation uncertainty. Thus, by varying  $\tau_W$ , one can change the value of  $\Lambda_{TP}$  and consequently, the value of the thermocreep  $\Lambda_{PT}$ . This indicates that a dependence of thermocreep on the temperature distribution along the channel walls exists, although it is weak. In problem No. 2 this dependence appears to a greater degree. At L = 5 and  $\delta$  = 2 the difference between the coefficients  $\Lambda_{PT}$  and  $\Lambda_{PT}$  comprises 14%, and increases with increase in  $\delta$ .

It is evident from Fig. 2b that the coefficient  $\Lambda_{TT}$  depends significantly on channel length. For small lengths L and large  $\delta$  this coefficient differs in problems Nos. 1 and 2. On the other hand, the coefficient  $\Lambda_{\widetilde{T}\widetilde{T}}$  depends weakly on channel length and differs only slightly between problems Nos. 1 and 2.

TABLE 1. TPD Index vs Number  $\delta$ 

ô	L=1		5	
	τ <sub>w</sub>	$\tilde{r}_{\omega}$	τω	$\tilde{\tau_w}$
$0 \\ 0,02 \\ 0,04 \\ 0,1 \\ 0,2 \\ 0,4 \\ 1 \\ 2$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	0,5 0,488 0,480 0,456 0,425 0,376 0,284 0,195	$\begin{array}{c} 0,5\\ 0,486\\ 0,475\\ 0,443\\ 0,408\\ 0,353\\ 0,259\\ 0,181 \end{array}$	$\begin{array}{c} 0,5\\ 0,486\\ 0,475\\ 0,444\\ 0,409\\ 0,352\\ 0,255\\ 0,176\end{array}$

The coefficients  $\Lambda_{T\widetilde{T}}$ ,  $\Lambda_{\widetilde{T}T}$  are not shown on the graph. Their values themselves are not of great practical interest, but their equality to each other is an additional criterion of the accuracy of the solution. The dependence of  $\Lambda_{T\widetilde{T}}$  on number  $\delta$  is analogous to that of  $\Lambda_{T\widetilde{T}}$ .

Thus, presence or absence of a preinput region has a significant effect on the value of the kinetic coefficients  $\Lambda_{PP}$ ,  $\Lambda_{TT}$  and  $\Lambda_{TT} = \Lambda_{TT}$ .

Figure 3 shows profiles of gas velocity in various channel sections at L = 5 and  $\delta$  = 2 for problem No. 1. The velocities  $u_x^P$  and  $u_x^T$  show slight changes. The velocity  $u_x^T$ , produced by the stepped temperature distribution, changes significantly. In the central channel section the velocity  $u_x^T$  has a maximum near the wall and a minimum in the center of the channel. With decrease in channel length and number  $\delta$  all three profiles equalize. In problem No. 2 the profiles are analogous to problem No. 1, therefore they are not shown graphically.

We will now consider the effect of thermomolecular pressure difference (TPD). Let the vessels which the channel connects have quite large, but finite volumes. Let different temperatures  $T_1$  and  $T_2$  be maintained in the vessels with some temperature distribution on the channel walls. Then aside from the heat flux there develops in the system a mass creep - thermocreep, directed from the "cold" vessel to the "hot." Since satisfaction of nonpenetration conditions on the vessel walls is assumed, i.e., the absence of drains, the thermocreep leads to an increase in pressure in the "hot" vessel and decrease therein in the "cold" one. This in turn stimulates a Poiseuille flow. The pressure difference will increase until the Poiseuille flow compensates the thermocreep. The system then reaches a steady state. In this case the pressure in the vessels  $P_1$  and  $P_2$  will be related to the temperatures  $T_1$  and  $T_2$  by the expression

$$\frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^{\gamma} \, . \label{eq:P1}$$

Making use of the smallness of the pressure and temperature heads, for the TPD index it is simple to obtain the expression  $\gamma = -\Lambda_{PT}/\Lambda_{PP}$ . In the free molecular regime ( $\delta = 0$ ) with diffuse scattering in channels of any length, any section, and with any temperature distribution on the channel walls the TPD index is equal to 0.5. We note that in experiments in round channels [6] in the free molecular regime deviations from 0.5 were observed. This can be explained by nondiffuse reflection of molecules in the experiments performed [6].

Table 1 shows numerical values of the TPD index, from which it is evident that its value decreases with increase in number  $\delta$  and channel length L. There is practically no difference in  $\gamma$  values for the different temperature distributions along the channel walls. This is related to the weak dependence of thermocreep  $\Lambda_{\text{PT}}$  upon  $\tau_{\text{W}}$ . But, as was shown above, this dependence does exist, and thus, there is also a dependence of the TPD index on temperature distribution along the channel walls.

Analysis of the temperature and pressure fields shows that in the range of numbers  $\delta$  considered the following inequalities are valid:

 $|\boldsymbol{\vartheta}^{T}| \leqslant 0.02; \quad |\boldsymbol{\vartheta}^{\widetilde{T}}| \leqslant 0.1; \quad |\boldsymbol{\tau}^{P}| \leqslant 0.05,$ 

where the quantities  $\vartheta^{T}$ ,  $\vartheta^{T}$ , and  $\tau^{P}$  are defined in Eq. (7). This means that gas motion under the influence of a temperature head can to a high degree of accuracy be regarded as

isobaric, while gas motion under the action of a pressure head may be considered isothermal. Without performing calculations, this conclusion is not obvious for a rarefied gas.

### NOTATION

 $\ell$ , channel length; *a*, channel height; P, pressure; T, temperature; f, distribution function; m, molecular mass; k, Boltzmann's constant; v, molecular velocity; x, y, z, coordinates;  $\tau_W$ , relative wall temperature; c, dimensionless molecular velocity; L =  $\ell/a$ , dimensionless channel length;  $\eta$ , dynamic gas viscosity; w, dimensionless thermal molecular velocity; u, gas velocity; n, gas density; q, thermal flux density; J<sub>k</sub>, thermodynamic flux;  $\Lambda_{\rm kn}$ , kinetic coefficient; X<sub>k</sub>, thermodynamic force; h, disturbance function; L<sub>s</sub>h, collision operator;  $\vartheta$ , relative gas pressure;  $\tau$ , relative gas temperature;  $\delta$ , rarefaction parameter;  $\gamma$ , TPD index.

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## ACTION OF THE HALL EFFECT ON FLOW AND HEAT TRANSPORT

IN A CONDUCTIVE GAS FLOW NEAR A ROTATING DISK

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The action of the tensor character of medium conductivity upon flow and heat transport in the boundary layer on a rotating disk is studied in the presence of an axial magnetic field and various directions of the angular velocity vector.

In an analysis of an MHD-boundary layer on a rotating disk [1] proposed an approximate method for integrating the nonlinear equations of motions, involving averaging of inertial terms over layer thickness. A modification of that method was later successfully used to calculate hydrodynamic and thermal boundary layers near a rotating disk with exhaust and draft of the medium through the porous surface of the body flowed over in the presence or absence of an external magnetic field [2-5]. Comparison of the moments of the friction forces and thermal fluxes calculated on the basis of the approximate and numerical methods revealed good agreement. In the present study the method of partial consideration of inertial terms will be used to determine flow and heat transport in a flow of conductive gas in the boundary layer on a rotating disk in the case where Hall phenomena play a significant role.

We will consider the motion of a viscous conductive gaseous medium near an infinite dielectric disk rotating at constant angular velocity  $\omega$  about the z axis in an external homogeneous axial magnetic field **B**. Neglecting the induced magnetic field, the system of hydrodynamic equations of the boundary layer with consideration of electromagnetic forces has the form

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